THE ROLE OF VISUALISATION SKILLS IN MATHS PROBLEM SOLVING

An important aspect of visual thinking is the ability to perform spatial manipulations mentally and to use these manipulations in solving problems. When only two of the three dimensions are accessible for viewing, visual thinking plays an important role, and students who have trouble with this process have difficulty abstracting information on points, lines and surfaces (a low level spatial ability). Bertoline and Miller (1990) argue that there is sufficient evidence that spatial visualisation is difficult for a large number of students. This paper reviews some of the applicable studies in visual perception, particularly those utilising computer technology, and examines the spatial visualisation abilities in a group of university technology education students using a computer generated visual perception test. The effect of interactive computer tutorials on visualisation and response times was compared using a pretest/posttest pilot study. The results showed significant gains in visualisation ability of the experimental group when compared with a matched control group. These results are consistent with other computer-orientated studies in visualisation skills enhancement.

INTRODUCTION

The development of visualisation skills can be considered an important objective for curriculum, as research shows that the development of spatial visualisation is a vital component of success in a wide range of engineering, technical, mathematical and scientific professions (Bertoline & Miller 1990; Wiley 1990a). Spatial visualisation ability has also been related to high achievements in mathematics, chemistry, biology and science (Ben-Chaim, Lappan & Houang, 1986). Engineering graphics and other visual representations provide a framework for visual thinking, a powerful cognitive technique for investigating and clarifying a problem. An important aspect of visual thinking is the ability to perform spatial manipulations mentally and to use these manipulations in solving problems. These spatial manipulations are defined as spatial visualisation. It is important that individuals involved in design education, and in particular engineering graphics, have a high level of spatial visualisation skills so they can visualise how a three-dimensional object may look when presented in a two-dimensional medium. When only two of the three dimensions are accessible for viewing, visual thinking plays an important role, and students who have trouble with this process have difficulty abstracting information on points, lines and surfaces (a low level spatial ability). Bertoline and Miller (1990) argue that there is sufficient evidence that spatial visualisation is difficult for a large number of students.

A number of instruments have been developed to assess spatial abilities. Eliot's (1980) classification of frequently used spatial tests describes twelve different task categories, however Ben-Chaim et al. (1986) termed these task categories 'paper and pencil tests', involving 'hands on' applications. These tests were not suitable for this study because computer generated tests were required. The use of computers to enhance visualisation skills has been reported by several researchers (Burton (1989) and Waldron (1985)] and others, such Bertoline & Miller, (1990) have developed instruments to measure such skills, but there is no documented research that has used computers

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specifically as a means to both measure and develop spatial visualisation skills. This study reviews some of the applicable studies in visual perception, particularly those utilising computer technology, and examines the development of spatial visualisation abilities in a group of university technology education students using a computer generated visualisation tutorials and visual perception tests.

**VISUAL LEARNING THEORIES**

The dilemma of how we visualise our environment 3-dimensionally has attracted man's curiosity since the time of Aristotle and other ancient Greek philosophers. Many of the early (pre 1900) theories on visual perception concerned 'errors of sight' or 'visual illusions'. In reviewing some of these early theories Pastore (1971) identifies Descartes as the first to examine the facts of perception in detail. His theories have influenced the formulation of all major theories of perception from the seventeenth century to the present day. Research into visual perception has been conducted by many different disciplines, the majority being psychologists investigating spatial testing. Eliot and Smith (1983) identified three major phases in the development of spatial testing which have led to various theories and studies in spatial visualisation over the past ninety years.

The first phase (1901–1938) was an effort by psychologists to establish and identify the presence of a spatial factor. The second phase (1938–1961) tried to identify different spatial factors and how they varied from each other. The third and final phase, according to Eliot and Smith (1983), centred on studies determining the interrelation of spatial abilities with other abilities, and the discovery of various sources of variance in testing of spatial abilities. The next developments of importance to the area of visual perception were technical in nature.

Whereas most studies have concentrated on the neural-visual aspects of visual perceptions others, for example, Ben-Chaim et al. (1986) have indicated that there seems to be a strong developmental aspect to spatial visualisation skills, a development that is not solely determined by age. Wiley's (1990a and 1990b) theories of visual learning provide perhaps the clearest conceptual framework for the development of such visual processes. He proposed a Hierarchy of Visual Learning that constitutes three primary stages, and within these primary stages are hierarchical steps or developmental stages that individuals pass through while on their way toward visual maturity (see Figure 1).

**Developing Visualisation Skills**

Wiley (1989) earlier proposed that one way to increase visual perception was to instigate a series of instructional steps that will lead a student from perceiving and drawing real 3-D objects, to perceiving and drawing multiviews and isometrics. The examples given in Figure 2 need not necessarily be hierarchical because, to an extent, visual perception is an innate capacity as well as a developmental process. An example of this is if a student has the innate ability to perceive the relationship between 2-D and 3-D drawings, there is no need to start from step 1.

Wiley (1990b) later proposed a computer graphics model for developing visual perception; this model involves seven conceptual steps (Figure 3). Throughout each computer-generated step, the student can have full control of the perceptual and electronic drawing process. At any point the process can be slowed or reversed to reinforce perceptual learning experiences. This model places emphasis on realistic representations of 3-D objects early within the learning sequence. In the model suggested, wireframe models follow, rather than precede, solid models as Wiley suggests that the ambiguities of wireframes are resolved by providing prior experience with related and easier to perceive rendered objects.

**1) Visual Cognition**

- Visual perception
- Visual memory
- Visualisation

**2) Visual Production**

- Externalisation
- Transmission
- Reception

**3) Visual Resolve**

- Comprehension or resolution of visual products

*Figure 1 Wiley's hierarchy of visual learning*

*Figure 2 Steps in Development of Visual Perception (Wiley, 1990a)*
The rapidly expanding graphics capability of computers has enabled users to create, modify, manipulate and ‘simulate’ 3-D models and thus allows the creation of new instructional methods for assisting students in developing their visual and spatial thinking abilities. Objects can be viewed both pictorially and orthographically simultaneously, edited quickly, and animated for visual comparisons. A number of such studies have used the computer in developing visualisation skills. Waldron (1985) conducted research using a computer graphics tutorial to teach orthographic projection (see Figure 4). The tutorial was designed so that the user may choose any point, line or surface to be represented in the three views or, the lines in any or all views may be selected. For instance, the selection of surface 5 produces the corresponding coloured surface in all three views. Similarly the selection of surface 3 produces the appropriate surface in another colour in all three views (see Figure 4). The results of her research indicated that the tutorial package contributed to increased student understanding of objects in the orthographic projection process. She proposed that if students interactively observe the points, lines and surfaces in the three principal views, they would be better able to develop a visualisation algorithm for themselves (see Figure 4).

Similarly Anand, Aziz and Agrawal (1987) developed a software program using 3-D graphics to improve visualisation skills. These skills were reinforced through two stages of user interaction, firstly through demonstration exercises, then through general routines allowing the creation of 3-D objects from data supplied by the user. A HyperCard application was developed by Burton (1989) to teach visual perception utilising orthographic projection. His tutorial illustrated the perceptual spatial rotations of the plane of projection from a typical pictorial to a planar graphic environment. The instructional sequence was self-paced with an animation option, however, the subjects were able to override the animation and review any card for as long as required.

**Figure 3** Wiley’s computer learning model

**Figure 4** Examples from Waldron’s computer tutorial

**Table 1**

<table>
<thead>
<tr>
<th>1) REAL OBJECT (e.g. a cube)</th>
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<tr>
<td>2) SCANNED OBJECT (e.g. the cube is scanned into the computer)</td>
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<tr>
<td>3) 3-D SHADED SOLID-MODEL ANIMATION (e.g. a computer-generated 3-D rendered example of the cube which can be manipulated through a 3-D plane)</td>
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<tr>
<td>4) STATIC 3-D WIRE-FRAME MODEL (e.g. a static computer-generated 3-D non-rendered example of the cube with all construction lines shown)</td>
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<tr>
<td>5) 3-D ISOMETRIC (e.g. a 3-D drawing system using two inclined 30° axes)</td>
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<tr>
<td>6) 2-D ORTHOGRAPHIC (e.g. a 2-D drawing system using 2-D drawings to show the top, side and front views of the cube, also known as multiview drawings)</td>
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<tr>
<td>7) 3-D AXONOMETRIC (e.g. a pictorial drawing of the cube with receding, but not converging, axes)</td>
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**Development of the Visualisation Tutorials**

Software was developed by the authors to provide an interactive, self-paced learning environment for individual users to explore orthographic projection. Visualisation tutorials were developed primarily to assist individuals having low spatial ability utilising a combination of 3-dimensional graphics and orthographic projection. Three-dimensional graphics tutorials were created using a CAD application, imported into a graphics application for further editing then imported into a HyperCard stack. Orthographic projections were all created using HyperCard’s own graphics package.

The first tutorial explained how to visualise three principal orthographic planes (front, top and right elevation). The second tutorial was similar, but included the animation of an object rotating through the three principal planes, as well as a self-paced option. The third tutorial utilised the concept of an object (in this case a computer) placed in the centre of a glass cube, with the six principal
Using Computers to Teach and Assess Spatial Visualisation Skills

Figure 5 Glass box projection

Figure 6 Example from exercise 4

Figure 7 Isometric view with requested plane

Figure 8 Screen display of possible answers

Orthographic planes being projected onto all six sides of the glass cube (see Figure 5). An animation was included which showed the cube being unfolded and revealing the six orthographic planes.

Four exercises were developed to indicate how to visualise the three principal orthographic planes. These exercises invited the subject to select the correct views from a number of answers, the program proceeding onto the next object when the correct view had been selected. The final exercise asked the subjects to choose the correct third angle representation of a given pictorial object (see Figure 6). Exercise four also included an on-line help, which accessed an explanation on how to view the orthographic planes. The instructional tutorials and exercises took approximately 30 to 45 minutes to complete.

Measurement of Spatial Visualisation Abilities

A number of instruments have been developed to assess spatial abilities. Eliot's (1980) 'paper and pencil' frequently used spatial tests have been mentioned previously but only one instrument, Bertoline and Miller's (1990) Visual Perception Test, appears to have been developed specifically for use on the computer. In this test an isometric object is displayed, and one of the six orthographic planes is listed below the object. At this point the subject must visualise how the requested view appears (see Figure 7). While the user is visualising the object, a timer records the amount of time from the initial display of the isometric view until the mouse button is clicked, this is recorded as visualisation time (VT). When the user can identify the given view the mouse button is clicked (on the object) and six possible answers appear in random order (see figure 8). Another timer is activated to record the reaction time (RT) from the display of possible answers until the choice has been made by placing the cursor on the desired selection and clicking the mouse. If an incorrect selection is made the word 'incorrect' is displayed, similarly, if the correct selection is made the word 'correct' is displayed and the software moves onto another object and view.

Thus this test measures both the time taken to visualise a 3-dimensional object and then the response time to choose a correct view from a display of six possible views. Visualisation time (VT), response time (RT) and responses from each section are logged, averaged and placed in separate fields for subsequent analysis (see Figure 9).

Methodology

Twenty-nine Technology Education students from the University of Tasmania were involved in a pilot study, which was conducted over a period of four weeks. The sample of 8 females and 21 males
between 18 and 44 years old consisted of twenty subjects in the experimental group, and nine subjects in the control group. All were familiar with the Macintosh computer and none had any previous experience with HyperCard. The subjects all had previous but limited experience with the concept of orthographic projection.

Both groups were exposed to the Visual Perception Test at the beginning and end of the four week test period whereas only the experimental group received the previously described Visualisation Tutorials and Exercises in the intervening period. The tests, tutorials and exercises were designed to be self-explanatory, interactive and self-paced. The subjects were informed of the sequence of the instructional tutorials and exercises, and that there would be no external interference while they were undertaking this stage of the study. The tutorials and exercises took approximately 30 to 45 minutes to complete.

**RESULTS AND DISCUSSION**

Mean visualisation (VT) and response (RT) times for both the experimental and control groups as tabulated below were compared by t-test for significant differences. Results indicate significant improvement (p<.0001) in both visualisation and response times between the pre and post-tests for the experimental group whereas there was no significant improvement for the control group in either visualisation or response times. When compared to the control group the experimental group showed significant post-test improvement in visualisation time (p<.005) and response time (p<.05).

It is of interest to note that most improvement following the visualisation and tutorial exercises occurred in the visualisation times rather than the response times suggesting that it is the spatial visualisation comprehension that is the rate limiting factor rather than the decision making process that follows. Interestingly the control group showed only marginal improvements in the post-test times suggesting the test itself achieved little improvement in visualisation skills (see Table 1).

These results are consistent with other computer-orientated studies carried out by Anand and others (1987), Burton (1989) and Waldron (1985) which also showed that increased exposure to different objects when developing visual skills does benefit students. The visualisation and tutorial exercises developed for this study, which were based on Wiley’s (1990a) computer graphics model, appear to have been equally as effective in enhancing spatial visualisation skills although it could not be claimed that they are more effective than paper and pencil equivalents. Further studies employing the Visual Perception Test would need to be performed to compare the effectiveness of computer-orientated instruction to traditional manual drafting methods. There have been previous studies conducted in this area (Foster 1987; Resetarits 1989), but none employed the methodology used in this study. Similarly the spatial visualisation abilities of different professions, such as architects, engineers and artists could be compared. The Visual Perceptions Test could also be modified for research into pre-adolescent and adolescent children’s spatial visualisation abilities, thus opening up new areas for research.

**CONCLUSION**

The major aim of this study was to investigate the effectiveness of computers as an instructional medium for teaching spatial visualisation skills. The results have shown that by implementing suitable instructional methods, visualisation skills of individuals can be improved and the implications for design education in particular are far reaching in that graphical representation is a fundamental communication tool that is...
used to transform conceptualised information into pictorial and orthographic views. By using tutorials similar to those utilised in this study, early ambiguities involving pictorial and orthographic views may be overcome, thus increasing the student's spatial awareness and subsequent success in design-based disciplines.

By using a similar methodology, investigations into other aspects of visual learning may be conducted, particularly for other areas of the curriculum than design education and other age groups than tertiary students. It is not suggested that computer based instruction and assessment are a universal panacea for visual learning, or even that they should replace the well tried traditional methods but rather that they can be successfully used as another medium, giving both the teacher and the learner another worthwhile and effective option.

References