Interactive versus observational

learning of spatial visualization of geometric transformations

ABSTRACT

This study compared interaction with a computer vs. observation as learning situations for low and high ability students' learning of spatial visualization and geometric transformations. Thirty-two fifth grade boys took the Differential Aptitude Test, Space Relations Subset (DAT), and then participated in the experiment. Pre-test and post-test were static spatial visualization problems using polyominos. During treatment, subjects were paired, one interactively solving computer-based polyomino puzzles, the other observing a yoked monitor in a separate room. Think-aloud protocol was used throughout. Overall, no significant differences were found between the two conditions. However, high achievers on the DAT benefited significantly more from observing (p.5). Low achievers benefited marginally more from the interactive condition, showing an increased incidence of holistic mental rotation strategy following the interactive condition.

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Introduction

Spatial visualization, defined as the ability to solve multi-step problems involving configurations of spatial elements, is an important factor in student success in mathematics and especially in geometry (Clements & Battista, 1992; Battista, 1990). Children's performance on both spatial tasks and geometry can improve significantly through appropriate use of manipulative tools (Fuys, Geddes, & Tischler, 1988).

More recently, interactive computer environments have opened up new possibilities for teaching geometry. Student use of computer programs, such as the Geometric Supposer, the Geometer's Sketchpad and Geometry Tutor, may sometimes improve learning of geometry (Yerushalmy & Chazan, 1990; Shelton, 1985; Anderson, Boyle & Reiser, 1985). Similarly, interactive computer graphics (in computer game-like scenarios) can improve spatial abilities (Larish & Anderson, 1995; Okagaki & Frensch, 1994; Subrahmanyam & Greenfield, 1994; Gagnon, 1985). However, relatively more passive observation has some advantages over interactive learning (Sweller, 1994). When students are engaged in a goal-oriented hands-on activity, the physical manipulation may occupy some of their mental resources, diverting attention away from focusing explicitly on the meaningful principles embodied in problems. Students may use means-end processing, i.e., solve the problem in ways that circumvent the academic content of the problems. Therefore, how interaction versus observation compares as learning situations deserves closer attention.

Interactive and observational training can be conceived of as differing levels of active control. Active control, defined as a situation where the subject physically manipulates the spatial situation relevant to some task, appears sporadically in research on spatial ability. Piaget & Inhelder (1948) and Whitcraft (1971) have suggested that active control is instrumental in the development of spatial abilities and the learning of spatial tasks. Active control, in the context of manipulatives and interactive computer graphics, can also improve existing spatial abilities (Gibson, 1962; Rovet, 1983; Gagnon, 1985; Embretson, 1987; Okagaki & Frensch, 1994; Subrahmanyam & Greenfield, 1994).

As mentioned earlier however, less active control, i.e., observation without hands-on interaction, may impose relatively less cognitive load on students, allowing them to better focus on the meaningful content of problems (Sweller, 1994).

So far, it is unclear how minimizing extraneous cognitive load impacts acquisition of spatial visualization skills. In other words, it is unclear how high and low active control compare as learning situations for spatial

visualization. Therefore, one purpose of the current study was to compare the effects of high and low active control on spatial visualization of geometric transformations.

Successful performance of spatial visualization tasks involves flexibility in selecting the optimal strategy for each item (Kyllonen, Woltz, & Lohman, 1981). Roughly speaking, there are three types of strategies used on the types of problems in standardized tests of spatial visualization (Schultz, 1991). These are; 1) analyze in terms of key features, i.e., making a decision based on verification that some key portion of a shape, present in one stimulus, is also present in another, 2) imagined movement of the viewer with reference to the object (usually occurring with a stimulus assumed to be larger than the subject, i.e., landscapes), and 3) 'imagined movement of the object', for example holistic mental rotation. Holistic mental rotation involves the representation of the entire shape of an object in a way that has something in common with the process of perceiving an external object physically rotating into congruence with another similar object (Shepard & Cooper, 1982). These strategies may also be combined in a variety of ways and/or also combined with less spatial types of reasoning such as deduction and verbal formulations.

The relationship between improvement in spatial visualization and strategy selection is an important question for this study. For example, when high active control situations facilitate improvements in spatial visualization, what are the changes in spatial visualization strategy? Are these improvements the result of changes in strategy? Low active control situations may also facilitate improvements in spatial visualization. Therefore another important question is, 'in cases where students experiencing a low active control situation make improvement in spatial visualization performance, what are the changes in spatial visualization strategy?'

Additionally there is reason to believe that spatially skilled students may respond differently to active versus observational situations from less spatially skilled students. The ability to flexibly use strategies implies a full repertoire of spatial visualization strategies. It is quite possible that some students are less skilled in spatial visualization because they have not mastered certain vital strategies, such as holistic mental rotation. Active versus passive situations are likely differentially advantageous for developing particular strategies. It seems quite logical that high active control activities may encourage strategies involving holistic mental imagery, since there are cognitive similarities between visual perception and mental imagery (Farah, 1985; Farah, Peronnet, Gonon, & Giard, 1988; Shepard & Cooper, 1982). Given the cognitive and structural similarities between the two processes (both involve transforming and matching), we hypothesize that active perception processes provide models at a cognitive level for mental imagery representations of the same processes.

In sum, this study aimed at comparing the effect of interaction with a computer and relatively more passive observation as learning situations for spatial visualization tasks involving geometric transformations on low and high spatial ability students. Specifically, the goal of this study was to set up high and low active control situations, look for improvements in spatial visualization skills and then investigate how these improvements related to differences in strategy selection of students with different spatial abilities.

METHOD

Participants

Participants of the study were 32 fifth-grade boys with a mean age of 11 years. Only one gender was included because it was felt that isolating the effects of active control on spatial visualization strategy would be easier without the addition of another grouping variable. If boys and girls have different levels of experience with computer games and different spatial visualization strategies, those differences might obscure the effects of active control.

The participants were recruited in a public elementary school district in the greater Phoenix metropolitan area. All participation in the study took place within the school. Participation was entirely on a voluntary basis.

Materials

To account for high and low spatial achievers, a standardized test of spatial visualization, the Differential Aptitude Test, Space Relations Subset (DAT) was administered prior to the experiment. Items on the differential aptitudes test are surface development problems, as shown in Figure 1.

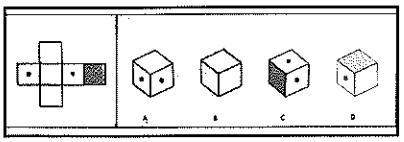


Figure 1. A sample of spatial folding items from a standard test of spatial visualization, the Differential Aptitude Test, Spatial Relations Subset

Two types of computer-based polyomino problems were used: 1) 'I-puzzles': Interactive puzzles where participants drag and rotate pieces (via mouse) to assemble them into a target shape (see Figures 2 and 3), 2) 'V-puzzles': static 'Visualization puzzles' where pieces cannot be moved, but participants were asked to determine if the puzzle was solvable. Any movements or rotations needed to solve V-puzzles must happen via spatial visualization in the mind of the participant (see Figure 4).

Polyomino problems were used as stimuli because they are both spatial visualization and transformational geometry problems. Polyomino problems are mathematical problems involving the assembly of a group of smaller shapes onto a larger target shape (see Figure 2). Both the smaller shapes and the larger target shape are composed of squares. The polyominos can be classified by the number of squares they contain. Thus, dominos contain two squares. Triominos contain three squares and Quatrominoes four squares, etc. The squares in polyominos are always arranged in a manner consistent with movement of a rook in chess, i.e., each square in a polyomino piece shares at least each one edge with another square in the same piece. Solving a polyomino problem (see Figure 3) means assembling all the polyomino pieces on the target shape such that the entire target shape is covered, none of the polyomino pieces overlap each other and no polyomino pieces hang off the target shape.

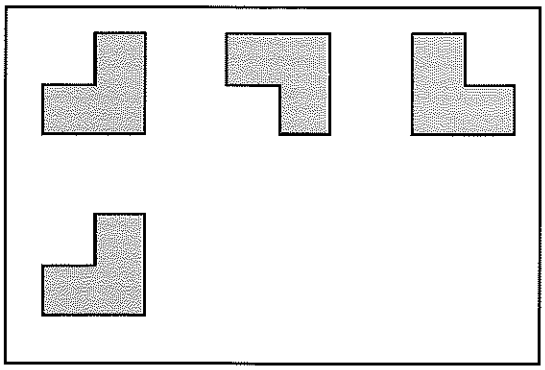


Figure 2. A polyomino puzzle (in the current study classified as an unsolved I-puzzle)

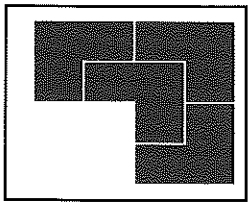


Figure 3. A solved polyomino puzzle (in the current study, classified as a solved I-puzzle)

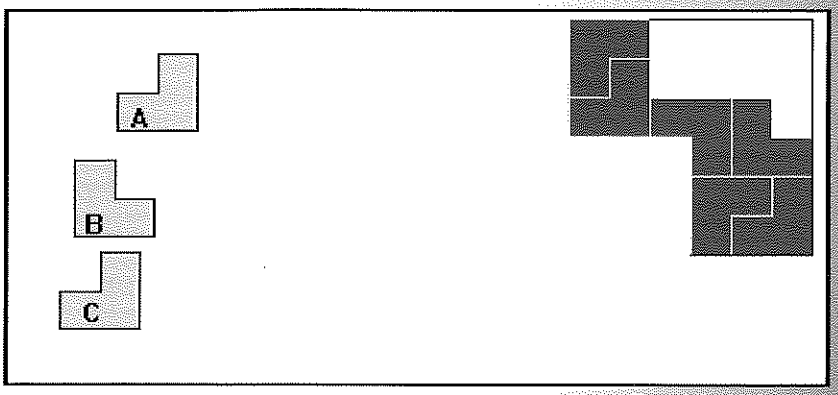


Figure 4. Can the puzzle be solved without removing the pieces already on the target grid?

Procedure

The experiment proper used a pre-test, treatment, post-test design. During the pre-test, participants thought aloud while solving four V-puzzles. In the treatment, participants were "yoked" together in "pilot-copilot" pairs. The pilot was instructed to think aloud, while inter-actively solving eight I-puzzles. The co-pilot was isolated in a separate room watching a monitor linked to the pilot's computer. The co-pilot saw exactly the same visual display, but was unable to interact in a hands-on manner (via mouse or any other input device) with the computer display. The co-pilot was instructed to pay close attention to the pilot's progress on the I-puzzles, and to think aloud. During post-test, pilot and co-pilot were separated and again asked to think aloud while solving four V-puzzles. All sessions were video and audiotaped.

For the data analysis, on the basis of the Differential Aptitude Test, Space Relations Subset (DAT), participants were classified as being high or low in spatial visualization skill. Forty percent of the participants (12) were classified as low in spatial visualization skill (low DAT) and sixty percent (20) were classified as high in spatial visualization skill. Since the DAT is intended for seventh graders at the lowest and our participants were fifth graders, the DAT provided a convenient way to dichotomize the sample into low and high spatial visualization achievers. The lower forty percent of the participants scored no better than what would be expected to be achieved by random guessing on the DAT. The pilot-copilot pairs were not matched in any particular way in terms of high/low DAT. Since the effects of low-low, high-high, or low-high pairings were not known, it seemed best to use random assignment.

Since the study focused on the strategies in addition to performance, a think-aloud protocol method was employed (e.g., Erickson and Simon, 1993; Anderson, 1987), i.e., the participant thinks aloud while engaged in problem solving. The think-aloud method is the simplest and most direct method of analyzing the contents of a subject's working memory while they solve problems (Anderson, 1987).

Analysis

Video tapes and think-aloud protocols were transcribed, encoded and analyzed using the principles of protocol analysis (Erickson & Simon, 1993; Anderson, 1987). Protocol analysis is comprised roughly of three steps: 1) the recorded data is transcribed, 2) the transcribed data is encoded, i.e., each verbalization is converted into a symbol representing a category of verbalization. For instance in the current study, each mention of turning or rotating a piece was converted to the symbol 'r' and 3) the encoded data is analyzed. Various frequencies can be done. Analysis of encoded data usually reveals patterns in the problem-solving process not readily apparent in the raw data.

During their review of videotapes, particular attention was paid to strategy type as indicated by hand gestures. For example, holistic mental rotation was often indicated by a gesture of thumb and forefinger rotating as if screwing on the lid of a jar (Just & Carpenter, 1985). By contrast, rotation by key feature (or decomposition) was indicated by gestures tracing out key feature of a shape, first on the original shape and then in a different orientation on a destination location on the target grid.

Gestures were also dichotomized as either pointing towards the source shape or the destination target grid. Source gestures, where a participant pointed to pieces of the puzzle that were not on the target grid, were used as evidence of the mental process of encoding, theorized to be associated with holistic mental rotation (Shepard & Cooper, 1982).

Statistical analyses included t-tests comparing pilots and copilots results on the pre-test, post-test and progress from pre-test to post-test. The data from the experiment was analyzed first using all the participants (both high and low DAT participants) for overall effects. A second analysis looked at the high and low DAT groups separately.

Additionally, an exploratory data analysis was also performed to investigate if there were any more complicated relationships between the variables not revealed in the main analyses. Using an interactive software package (DataDesk), various graphical displays of the data were interactively manipulated and explored to uncover any non-linear relationships present in the data. These techniques, pioneered by Tukey (Tukey, 1977), are common practice in exploratory studies.

Results

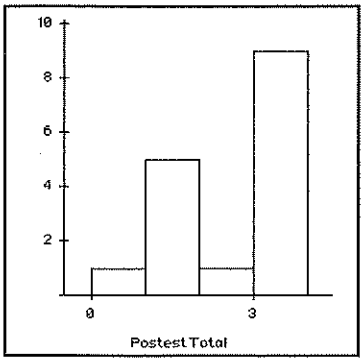
The means and standard deviations of pre and posttests for pilots and copilots are presented in Table 1. Although copilots improved slightly more than pilots, t-tests comparing the pilots' and copilots' pre-test and post-test results and pre-test to post-test progress did not indicate significant differences. In other words, neither high (pilot) nor low active control (co-pilot) situation was clearly superior as a learning environment for spatial visualization.

TABLE 1. Pre-test and post-test results for pilots and copilots

		Pretest		Posttest	
	N	Mean	SD	Mean	SD
Pilots	16	2.44	.892	2.188	.856
Copilots	16	2.06	.772	2.250	.981

Further exploratory data analysis however revealed some more complex relationships in the data. A histogram of the pilots' post-test scores revealed a distinctly bimodal distribution. See Figure 5. This indicated that for some students, active control was helpful in the acquisition of spatial visualization skills, while for others it was detrimental.

Figure 5. Histogram of pilots' post-test scores: The bars



represent, from left to right, the number of pilots who successfully solved a total of zero, one, two and three post-test puzzles. Thus, nine pilots solved a total of three post-test puzzles.

Analysis of the data as high and low DAT groups revealed some interesting results. Table 2 shows the results of this analysis. Low DAT pilots showed an increase in their scores, pre-test to post-test. However, low DAT copilots showed a decrease in scores from pre-test to post-test. A t-test was performed comparing low DAT pilots and copilots pre-test to post-test differences. These differences between low DAT pilots and copilots were not quite significant at the .05. A small group size (n=12) may have made a significance of .05 less likely. The level of significance actually obtained was .1. It seems quite possible that these numbers indicate a trend that might be more obvious had "n" been larger.

	Lower 40% of subjects on the DAT	Upper 60% of subjects on the DAT
Pilots	.375	-.875
Copilots	-.750	.500

TABLE 2. Means of post-test pre-test differences, grouped by performance on the DAT

Results of high DAT participants were more dramatic. High DAT pilots showed a decrease from pre-test to post-test. However, high DAT copilots showed an increase in scores. The differences between high DAT pilot and copilot performance were significant at the .05 level. A t-test comparing pilot and copilot pre-test to post-test differences obtained a two-tailed significance of .026, (n=20, t=2.458).

Summarizing the trends for high and low DAT groups: Within the high DAT group, copilots made progress, while the pilots regressed. Within the low DAT group, the pilots made progress while the copilots regressed.

Analysis of the think-aloud data and videotapes shed some light on the differences between high and low achievers on DAT. Of the six participants whose think aloud protocols were analyzed, four conformed to the trends found in the statistical analysis, i.e., there were two low DAT pilots who made pre-test to post-test progress and there were two high DAT copilots who made pre-test to post-test progress. Six participants were selected for think aloud analysis on the basis of having complete data and conforming with the trends found in the data. More protocols were not analyzed because pairs of participants were considered to be data units. Because of the complexity of the data gathering procedures (simultaneously operating one computer with two monitors, three video cameras, two audio tape recorders, and note taking – divided between two separate rooms) some data were lost. The participants whose think aloud protocols were analyzed were pairs with one hundred per cent complete data sets.

Table 3 summarizes the think aloud protocols. The column heading "Source" refers to gestures or other references to the shapes that are off the grid. "Destination" refers to gestures toward the target grid. Gestures toward source are often evidence of encoding behavior and thus evidence of holistic mental imagery or holistic mental rotation. In the column "Source/Destination" are the ratios of number of source gestures to destination gestures. Numbers in the column labeled "Key" are the number of instances of evidence of key-feature reasoning. "Holistic" refers to the number of instances of evidence of holistic reasoning. "Holistic/Key" refers to the ratio of instances of holistic rotation to instances of key reasoning.

		Source	Dest-ination	Source/ Dest-ination	Key	Holistic	Holistic/ Key
Copilot Z	Pretest	25	20	1.25	18	5	.278
	Posttest	7	7	1	3	1	.33
Copilot Y	Pretest	0	0	0/0	1	0	0
	Posttest	3	2	.66	0	1	1/0
Pilot Q	Pretest	2	9	.22	1	0	0
	Posttest	10	11	.9	4	6	1.5
Pilot S	Pretest	0	5	0	4	0	0
	Posttest	6	9	.67	7	1	.143

TABLE 3. Tabulation of coded data

Note from Table 3 that the two low scorers on the DAT who benefited from the active control situation, showed no evidence of using holistic mental rotation on the pre-test. On the post-test, however, these same participants (low scorers on the DAT who benefited from the active control situation) showed substantial use of holistic mental rotation. The ratio of source to destination also increased from pre-test to post-test (see Table 3). The increased use of holistic mental rotation coincided with an improved ability to mentally visualize configurations involving groups of two or three shapes.

By contrast, high scorers on the DAT, who benefited from the low active control condition, exhibited no discernible consistent changes in strategy. Evidently, their improvements resulted, not from changes in strategy, but through an improvement in performance of existing strategies.

Analysis of the think-aloud data of the treatments revealed that the copilots focused more on overall configurations, while the pilots focused more on the manipulation of individual shapes. The pilots' treatment protocols consisted almost entirely of a "blow-by-blow" account of their concurrent manipulation of shapes on the computer. By contrast, when copilots did focus on individual shapes, they suggested hypothetical transformations of shapes, i.e., where and how shapes should be moved and rotated.

DISCUSSION

Overall, no significant differences were found between high and low active control situations in learning spatial visualization of geometric transformations. However, high and low spatial ability students responded differently to the experimental situations. Low achievers gained more on the active control situations while high spatial achievers benefited more from relatively passive (observational) learning situations. There are some explanations for this.

On the experimental pre-test involving V-puzzles, the low DAT (40% of the) participants did not use strategies involving holistic mental rotation. They seemed not to have holistic mental rotation in their repertoire of spatial visualization strategies. Apparently, these participants did not have the cognitive resources and/or motivation to deal meaningfully with the surface development problems on the DAT. For these participants, the active control treatment allowed them to focus on the interactive transformation of individual shapes. The active control situation provided additional sense information (kinesthetic and proprioceptive) not present in the observational condition. While dragging or clicking the mouse, the pilot experienced kinesthetic sense information from his fingers, hand and arm. Similarly, while manipulating the mouse, the pilot received proprioceptive sense information reflecting changes in positioning of his joints. While manipulating the mouse, joints experiencing change in position are the joints of fingers, wrist and elbow. This kinesthetic and proprioceptive sense information would be correlated with the visual feedback coming from the computer screen. This additional sense information was perhaps encoded into an integrated memory of the transformation of the shape, facilitating the later creation of mental imagery used for holistic mental rotation. Thus, for some less skilled participants, the high active control condition encouraged the inclusion of holistic mental rotation in their repertoire of spatial visualization strategies. However, these less skilled participants were probably not sufficiently engaged by spatial visualization problems to benefit from the observational copilot condition.

By contrast, participants who achieved higher on the DAT (60% of the participants) already had a more complete repertoire of spatial visualization strategies. Apparently, they had the required cognitive resources or motivation to deal with these problems. They had little to gain from active control treatment and, in fact, the active control situation interfered with their subsequent performance on spatial visualization problems. The copilot condition, however, provided more skilled participants with a goal free situation where, unencumbered with interaction with the computer, they could focus on configurations and suggest hypothetical solutions.

Educational implications

From a practical perspective, this study suggests that the decision of whether to use manipulatives and interactive computer programs for subjects involving spatial visualization, such as geometry and mathematics, should be made on the basis of individual differences among students. Less skilled students who do not have a full repertoire of spatial visualization strategies may benefit from interactive situations, but may be bored with more passive observation. In contrast, for more advanced students, interactive activities may actually interfere with pre-existing cognitive structures. Observation of peers interacting with computer programs may provide skilled students with opportunities to contemplate configurations and to suggest hypothetical solutions. Observation of a peer may provide advantages over observing an expert, demonstrating solutions. Since students working problems may not proceed directly to solutions, observation of a peer provides ample opportunities for detecting dead-end configurations and suggesting possible

alternatives. A relatively less spatially skilled student interacting with a computer and a more spatially skilled peer observer may thus form a useful social learning unit.

In terms of practical implications one might consider 1) the classroom and 2) instructional design. In the classroom, an astute teacher quickly gets an intuitive feel for whether a student is struggling with some new material. Many students can be quite extroverted in their struggles, while others suffer silently and only betray their frustrations through subtle facial cues. In any case, if the content area has a highly spatial or shape oriented component, the teacher should consider having as resources several interactive computer programs where the interaction is integral with understanding the shapes, the relationships between shapes and the meaning directly connected to those shapes. So for example, if one is teaching geometry, there are a number of cleverly designed geometry programs (Geometer's Sketchpad, WinGeom, Cabri, etc.). A struggling student may greatly benefit with some theme-oriented interactive explorations or better yet some goal oriented tasks where interaction is part of the problem-solving. Students who are quicker picking up the material may solidify their gains by being paired as a consultant with the weaker students who is actually operating the program. In the long run, both students may benefit by alternating roles. These kinds of arrangements have other defacto benefits when there are less computers than students in the classroom.

It is interesting that these pedagogical approaches have also evolved on their own often without a formal knowledge of underlying cognitive principles. So for example, compass, triangles and rulers were long the mainstay of introductory geometry. Instructors of organic chemistry often have their students turn plastic or wooden models in their hands as an introduction to symmetry transformations. Only once the students have had a chance to internalize these operations, does the instructor expect them to be able to extrapolate the same from a two-dimensional diagram in a textbook. One hopes with a formal analysis of these principles, these types of pedagogy can be refined. Another point is that from a psychological and educational perspective, the distinction between hand-held manipulatives and interactive virtual objects in a computer program may sometimes be an artificial one (Olkun, 2003). Yet, each has its advantages (more tactile information versus cleverly designed constraints and interaction with more abstract objects), but the teacher should feel free to use whatever seems to be most germane to the educational situation in terms of providing educational interaction.

The principles outlined in this study also have implications for the instructional design of educational software. For example, one might design contextualized tutoring that is adaptive to the type of difficulties students encounter. If the student is struggling with something that seems to relate to shapes, then remedial exercises might involve interaction with those shapes. If the student is struggling with symbolic manipulation of some sort such as working equations, then a completely different form of remediation is called for. It may seem that it would be difficult to program this type of artificial intelligence into software. However, if one systematically categorizes the learning difficulties students have with a particular type of exercise, it may be possible to deduce the type learning difficulty from the mistake made. The think-aloud method discussed in this paper is one effective means for formative evaluation of software. Perhaps this type of educational software should be designed incrementally: 1) program the exercises, 2) analyze the type of learning difficulties associated with various types of mistakes, 3) design a rule-based adaptive tutoring system that provides the help that students need. With an involvement in this incremental design process, one would also gain knowledge about how people learn and how to teach, connecting the circle between designer, scientist and teacher.

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